CHAPTER 34
BEVEL AND HYPOID GEARS

Theodore J. Krenzer, M.S.
Director of Research and Development
Gleason Machine Division
Rochester, New York

Robert G. Hotchkiss, B.S.
Director, Gear Technology
Gleason Machine Division
Rochester, New York

34.1 INTRODUCTION

This chapter provides you with information necessary to design a bevel- or hypoid-gear set. It includes guidelines for selecting the type and size of a gear set to suit the application requirements. Equations and graphs are provided for calculating gear-tooth geometry, strength, surface durability, and bearing loads.

Although the text provides sufficient data to design a gear set, reference is also made to appropriate American Gear Manufacturer's Association (AGMA) publications and software available for computer-aided design.

34.2 TERMINOLOGY

34.2.1 Types of Bevel and Hypoid Gears

Straight-bevel gears are the simplest form of bevel gears. The teeth are straight and tapered, and if extended inward, they would pass through the point of intersection of the axes. See Fig. 34.1.

Spiral-bevel gears have teeth that are curved and oblique to their axes. The contact begins at one end of the tooth and progresses to the other. See Fig. 34.2.

Zerol bevel gears have teeth that are in the same general direction as straight-bevel gears and are curved similarly to spiral-bevel gears. See Fig. 34.3.

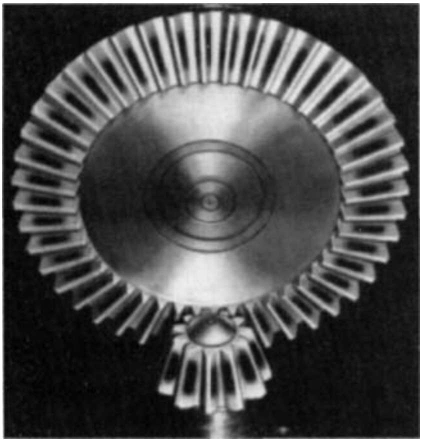

FIGURE 34.1 Straight-bevel set. *(Gleason Machine Division.)*

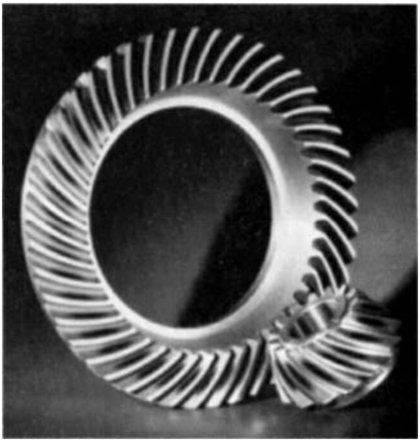

FIGURE 34.2 Spiral-bevel set. *(Gleason Machine Division.)*

Hypoid gears are similar in appearance to spiral-bevel gears. They differ from spiral-bevel gears in that the axis of the pinion is offset from the axis of the gear. See Fig. 34.4.

34.2.2 Tooth Geometry

The nomenclature used in this chapter relative to bevel and hypoid gears is illustrated in Figs. 34.5, 34.6, and 34.7.

The following terms are used to define the geometry:

Addendum of pinion (gear) a_p (a_G) is the height that the tooth projects above the pitch cone.

Backlash allowance B is the amount by which the circular tooth thicknesses are reduced to provide the necessary backlash in assembly.

Clearance c is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

Cone distance, mean A_m is the distance from the apex of the pitch cone to the middle of the face width.

Cone distance, outer A_o is the distance from the apex of the pitch cone to the outer ends of the teeth.

Control gear is the term adopted for bevel gearing in place of the term *master gear,* which implies a gear with all tooth specifications held to close tolerances.

Crown to crossing point on the pinion (gear) x_o (X_o) is the distance in an axial section from the crown to the crossing point, measured in an axial direction.

Cutter radius r_c is the nominal radius of the face-type cutter or cup-shaped grinding wheel that is used to cut or grind the spiral-bevel teeth.

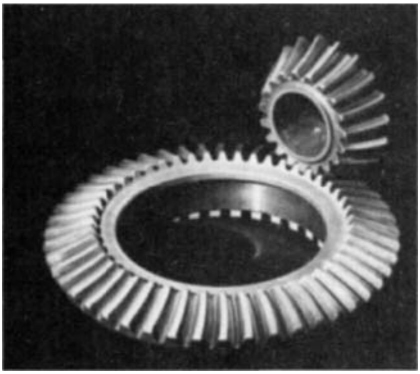

FIGURE 34.3 Zerol bevel set. *(Gleason Machine Division.)*

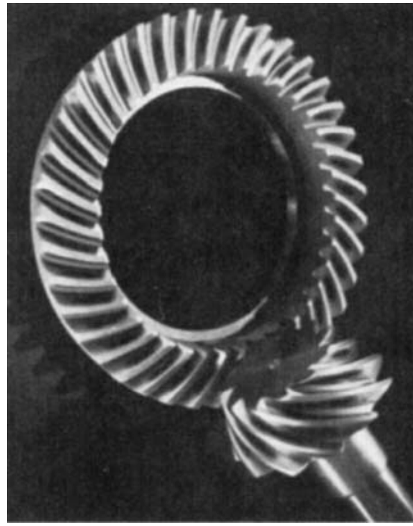

FIGURE 34.4 Hypoid set. *(Gleason Machine Division.)*

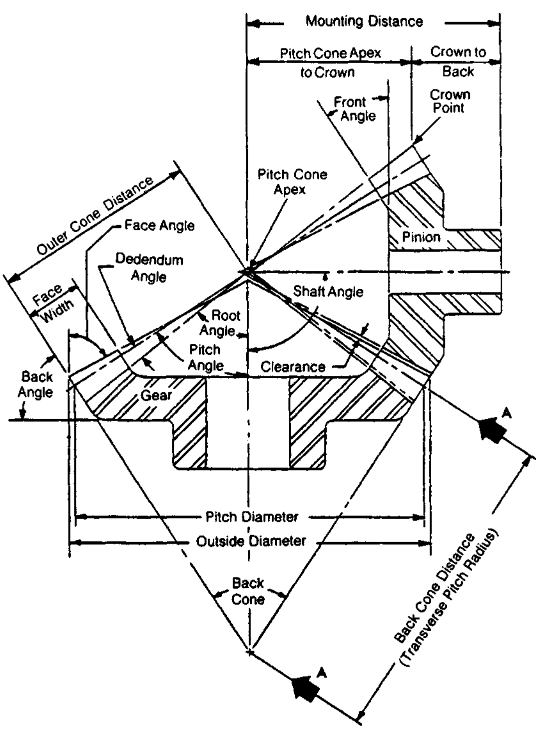

FIGURE 34.5 Bevel-gear nomenclature—axial plane. Section A-A is illustrated in Fig. 34.6.

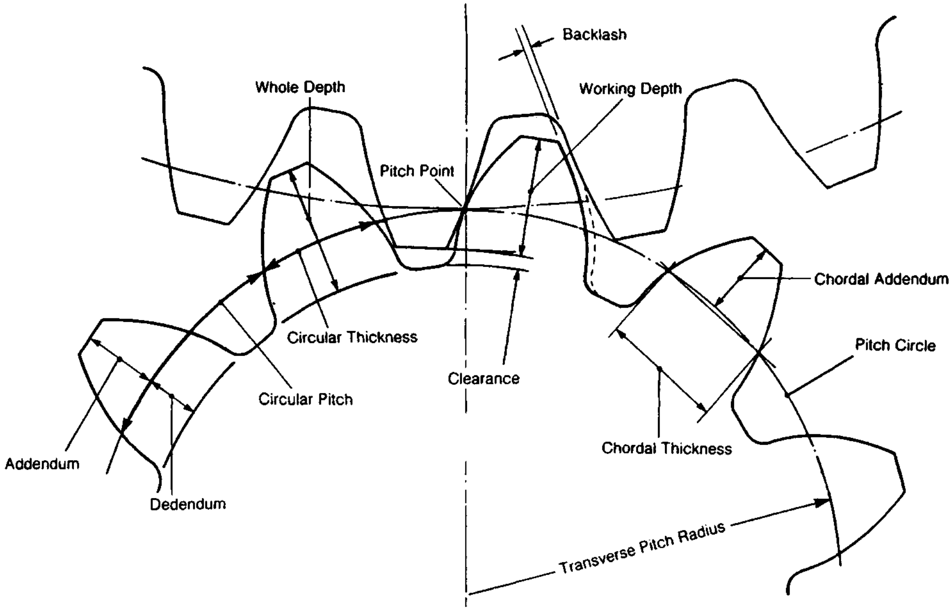

FIGURE 34.6 Bevel-gear nomenclature—mean transverse section AA in Fig. 34.5.

Dedendum angle of pinion (gear) δ_P (δ_G) is the angle between elements of the root cone and pitch cone.

Dedendum angles, sum of $\Sigma\delta$ is the sum of the pinion and gear dedendum angles.

Dedendum of pinion (gear) b_p (b_G) is the depth of the tooth space below the pitch cone.

Depth, mean whole h_m is the tooth depth at midface.

Depth, mean working h is the depth of engagement of two gears at midface.

Diametral pitch P_d is the number of gear teeth per unit of pitch diameter.

Face angle of pinion (gear) blank γ_o (Γ_o) is the angle between an element of the face cone and its axis.

Face apex beyond crossing point on the pinion (gear) G_o (Z_o) is the distance between the face apex and the crossing point on a bevel or hypoid set.

Face width F is the length of the teeth measured along a pitch-cone element.

Factor, mean addendum c_1 is the addendum modification factor.

Front crown to crossing point on the pinion (gear) x_i (X_i) is the distance in an axial section from the front crown to the crossing point, measured in the axial direction.

Hypoid offset E is the distance between two parallel planes, one containing the gear axis and the other containing the pinion axis of a hypoid-gear set.

Number of teeth in pinion (gear) n (N) is the number of teeth contained in the whole circumference of the pitch cone.

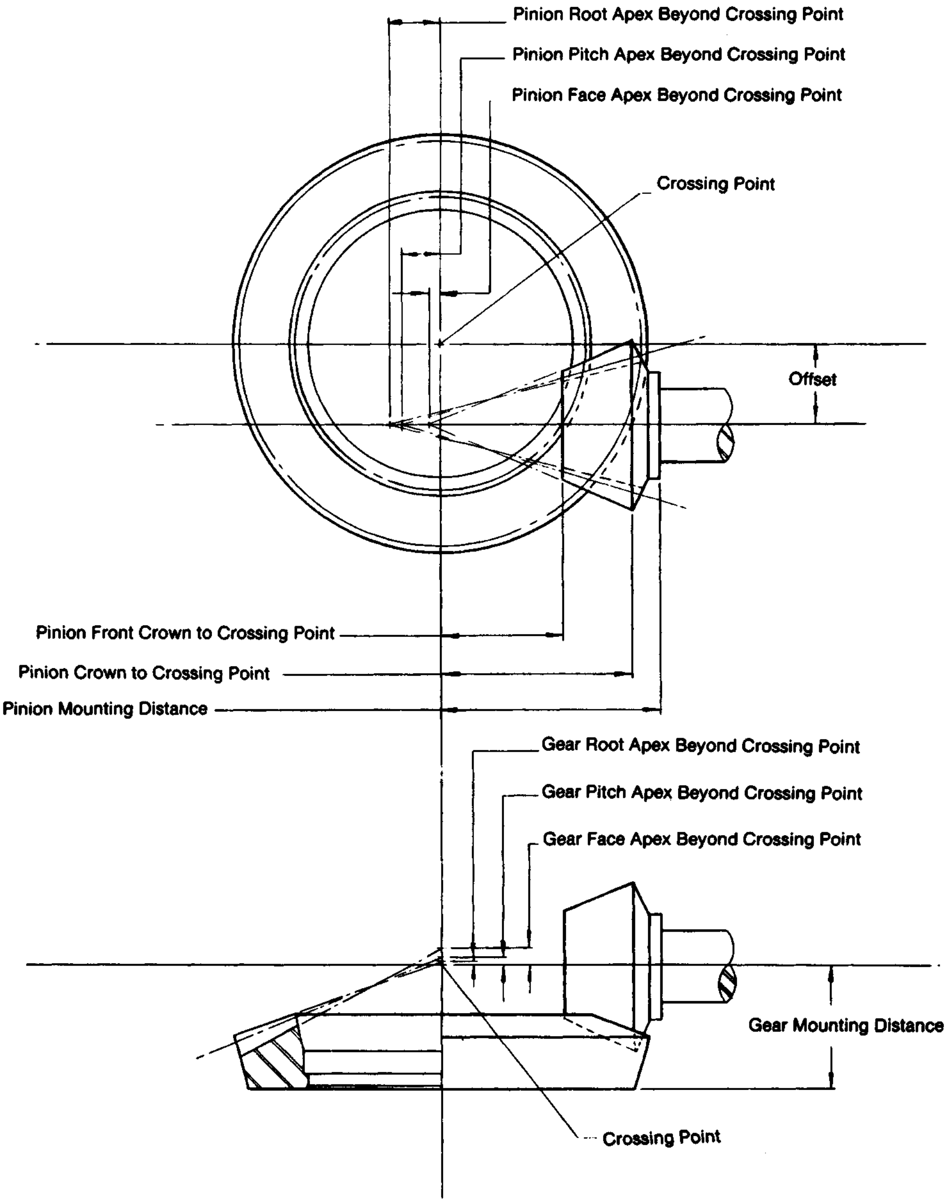

FIGURE 34.7 Hypoid gear nomenclature.

Pitch angle of pinion (gear) γ (Γ) is the angle between an element of the pitch cone and its axis.

Pitch apex beyond crossing point on the pinion (gear) G (Z) is the distance between the pitch apex and the crossing point on a hypoid set.

Pitch diameter of pinion (gear) d (D) is the diameter of the pitch cone at the outside of the blank.

Pitch, mean circular p_m is the distance along the pitch circle at the mean cone distance between corresponding profiles of adjacent teeth.

Pressure angle ϕ is the angle at the pitch point between the line of pressure which is normal to the tooth surface and the plane tangent to the pitch surface. It is specified at the mean cone distance.

Ratio, gear m_G is the ratio of the number of gear teeth to the number of pinion teeth.

Root angle of pinion (gear) γ_R (Γ_R) is the angle between an element of the root cone and its axis.

Root apex beyond crossing point on the pinion (gear) G_R (Z_R) is the distance between the root apex and the crossing point on a bevel or hypoid set.

Shaft angle Σ is the angle between the axes of the pinion shaft and the gear shaft.

Spiral angle ψ is the angle between the tooth trace and an element of the pitch cone. It is specified at the mean cone distance.

Spiral-bevel gear, left-hand is one in which the outer half of a tooth is inclined in the counterclockwise direction from the axial plane through the midpoint of the tooth, as viewed by an observer looking at the face of the gear.

Spiral-bevel gear, right-hand is one in which the outer half of a tooth is inclined in the clockwise direction from the axial plane through the midpoint of the tooth, as viewed by an observer looking at the face of the gear.

Tangential force W_t is the force applied to a gear tooth at the mean cone distance in a direction tangent to the pitch cone and normal to a pitch-cone element.

Thickness of pinion (gear), mean circular t (T) is the length of arc on the pitch cone between the two sides of the tooth at the mean cone distance.

Thickness of pinion (gear), mean normal chordal t_{nc} (T_{nc}) is the chordal thickness of the pinion tooth at the mean cone distance in a plane normal to the tooth trace.

34.2.3 Calculation Methods

Four methods of blank design are commonly used in the design of bevel and hypoid gears:

1. Standard taper
2. Duplex taper
3. Uniform taper
4. Tilted root-line taper

The taper you select depends in some instances on the manufacturing equipment available for producing the gear set. Therefore, before starting calculations, you should familiarize yourself with the equipment and method used by the gear manufacturer.

34.3 GEAR MANUFACTURING

34.3.1 Methods of Generation

Generation is the basic process in the manufacture of bevel and hypoid gears in that at least one member of every set must be generated. The theory of generation as applied to these gears involves an imaginary generating gear, which can be a crown gear, a mating gear, or some other bevel or hypoid gear. The gear blank or workpiece is positioned so that when it is rolled with the generating gear, the teeth of the workpiece are enveloped by the teeth of the generating gear.

In the actual production of the gear teeth, at least one tooth of the generating gear is described by the motion of the cutting tool or grinding wheel. The tool and its motion are carried on a rotatable machine member called a cradle, the axis of which is identical with the axis of the generating gear. The cradle and the workpiece roll together on their respective axes exactly as would the workpiece and the generating gear.

The lengthwise tooth curve of the generating gear is selected so that it is easily followed with a practical cutting tool and mechanical motion. Figure 34.8 illustrates the representation of a generating gear by a face-mill cutter. Figure 34.9 shows the basic machine elements of a bevel-gear face-mill generator.

Most generating gears are based on one of two fundamental concepts. The first is complementary crown gears, where two gears with 90° pitch angles fit together like mold castings. Each of the crown gears is the generating gear for one member of the mating set. Gears generated in this manner have line contact and are said to be *conjugate* to each other. With the second concept, the teeth of one member are form-cut without generation. This member becomes the generating gear for producing the mating member. Again, gears generated in this manner are conjugate to each other.

34.3.2 Localization of Contact

Any displacement in the nominal running position of either member of a mating conjugate gear set shifts the contact to the edges of the tooth. The result is concentrated loading and irregular motion. To accommodate assembly tolerances and deflections resulting from load, tooth surfaces are relieved in both the lengthwise and profile directions. The resulting localization of the contact pattern is achieved by using a generating setup which is deliberately modified from the conjugate generating gear.

34.3.3 Testing

The smoothness and quietness of operation, the tooth contact pattern, the tooth size, the surface finish, and appreciable runout can be checked in a running test. This is a subjective test. The machine consists of two spindles that can be set at the correct shaft angle, mounting distances, and offset. The gear to be inspected is mounted on

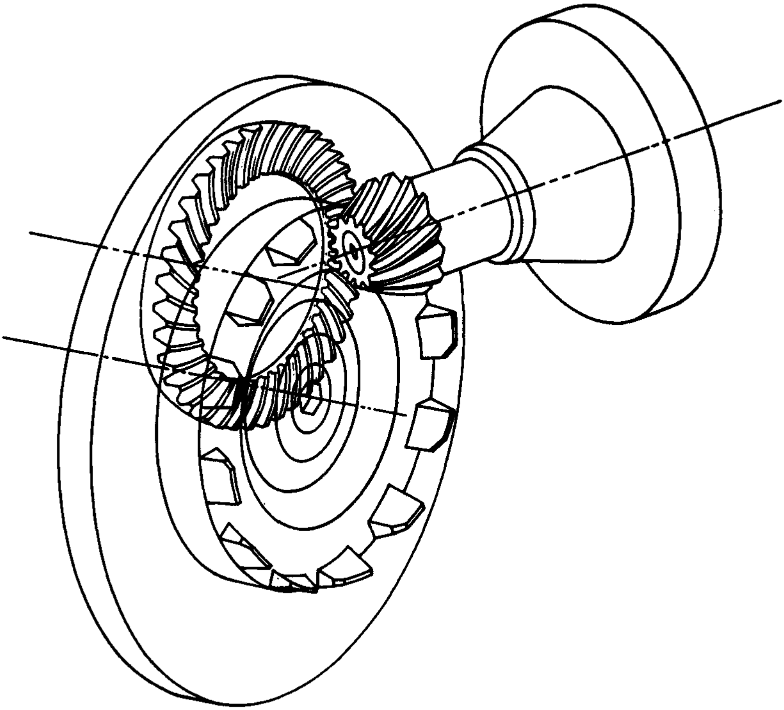

FIGURE 34.8 Imaginary generating gear.

one spindle, and the mating gear or a control gear is mounted on the other spindle. Tooth contact is evaluated by coating the teeth with a gear-marking compound and running the set under light load for a short time. At the same time, the smoothness of operation is observed. Spacing errors and runout are evaluated by noting variations in the contact pattern on the teeth around the blank. Poor surface finish shows up as variations within the marked contact pattern. Tooth size is measured by locking one member and rotating a tooth of the mating member within the slot to determine the backlash.

The contact pattern is shifted lengthwise along the tooth to the inside and outside of the blank by displacing one member along its axis and in the offset direction. The amount of displacement is used as a measure of the set's adjustability.

It is normal practice for tooth spacing and runout to be measured with an additional operation on inspection equipment designed specifically for that purpose. AGMA publication 390.03a specifies allowable tolerances for spacing and runout based on diametral pitch and pitch diameter.

Double- and single-flank test equipment can be used to measure tooth-profile errors, tooth spacing, and runout. The test equipment has transducers on the work spindles, and the output data are in chart form. The output data not only provide a record of the quality of the gear set, but can also be related to gear noise.

Three-dimensional coordinate-measuring machines can be used to compare the actual gear-tooth geometry with theoretical data.

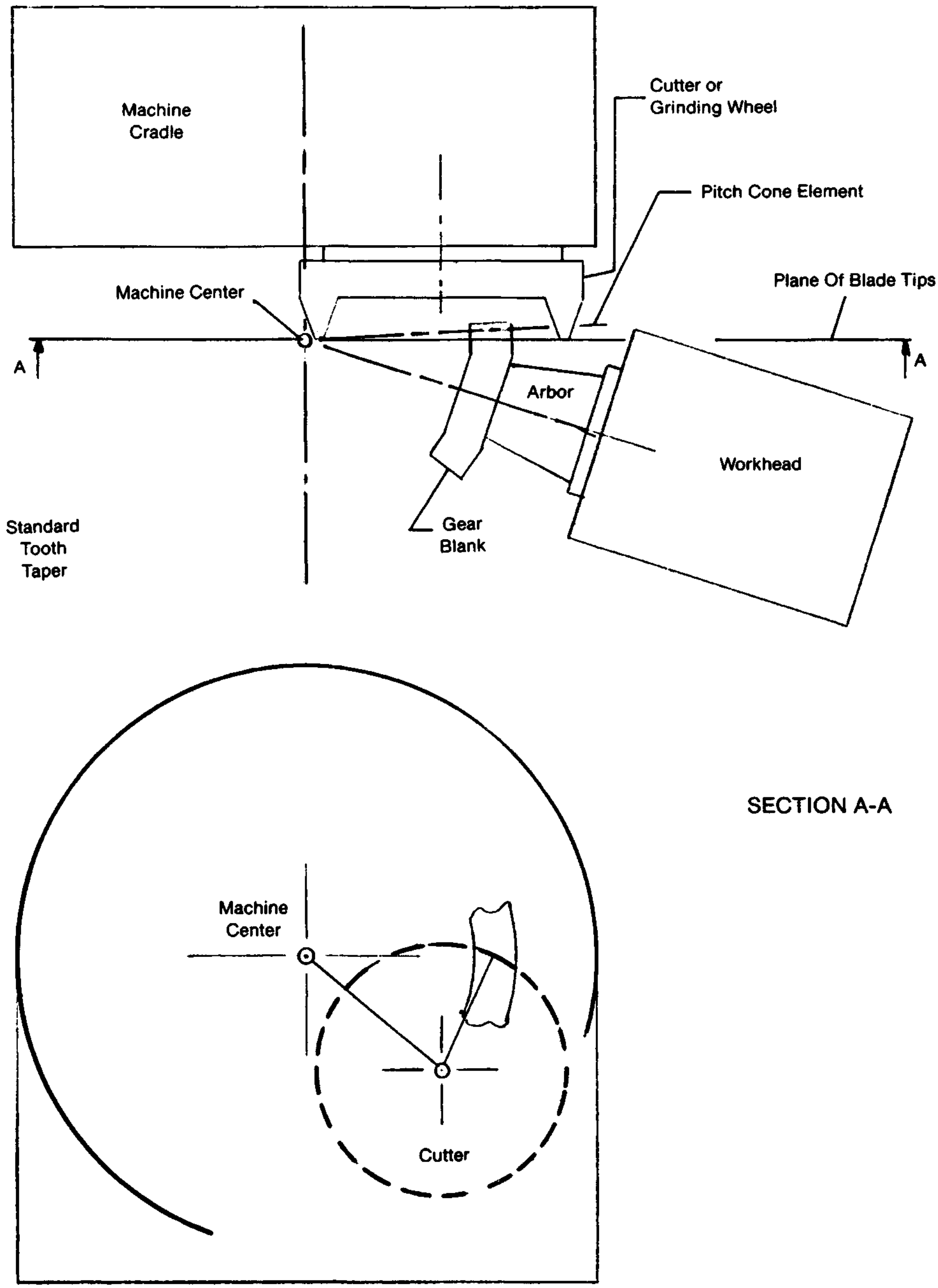

FIGURE 34.9 Basic machine setup of spiral-bevel face-mill generator.

34.4 GEAR DESIGN CONSIDERATIONS

34.4.1 Application Requirements

Bevel and hypoid gears are suitable for transmitting power between shafts at practically any angle and speed. The load, speed, and special operating conditions must be defined as the first step in designing a gear set for a specific application.

A basic load and a suitable factor encompassing protection from intermittent overloads, desired life, and safety are determined from

1. The power rating of the prime mover, its overload potential, and the uniformity of its output torque
2. The normal output loading, peak loads and their duration, and the possibility of stalling or severe loading at infrequent intervals
3. Inertia loads arising from acceleration or deceleration

The speed or speeds at which a gear set will operate must be known to determine inertia loads, velocity factor, type of gear required, accuracy requirements, design of mountings, and the type of lubrication.

Special operating conditions include

1. Noise-level limitations
2. High ambient temperature
3. Presence of corrosive elements
4. Abnormal dust or abrasive atmosphere
5. Extreme, repetitive shock loading or reversing
6. Operating under variable alignment
7. Gearing exposed to weather
8. Other conditions that may affect the operation of the set

34.4.2 Selection of Type of Gear

Straight-bevel gears are recommended for peripheral speeds up to 1000 feet per minute (ft/min) where maximum smoothness and quietness are not of prime importance. However, ground straight bevels have been successfully used at speeds up to 15 000 ft/min. Plain bearings may be used for radial and axial loads and usually result in a more compact and less expensive design. Since straight-bevel gears are the simplest to calculate, set up, and develop, they are ideal for small lots.

Spiral-bevel gears are recommended where peripheral speeds are in excess of 1000 ft/min or 1000 revolutions per minute (r/min). Motion is transmitted more smoothly and quietly than with straight-bevel gears. So spiral-bevel gears are preferred also for some lower-speed applications. Spiral bevels have greater load sharing, resulting from more than one tooth being in contact.

Zerol bevel gears have little axial thrust as compared to spiral-bevel gears and can be used in place of straight-bevel gears. The same qualities as defined under straight bevels apply to Zerol bevels. Because Zerol bevel gears are manufactured on the same equipment as spiral-bevel gears, Zerol bevel gears are preferred by some manufacturers. They are more easily ground because of the availability of bevel grinding equipment.

Hypoid gears are recommended where peripheral speeds are in excess of 1000 ft/min and the ultimate in smoothness and quietness is required. They are somewhat stronger than spiral bevels. Hypoids have lengthwise sliding action, which enhances the lapping operation but makes them slightly less efficient than spiral-bevel gears.

34.4.3 Estimated Gear Size

Figures 34.10 and 34.11 relate size of bevel and hypoid gears to gear torque, which should be taken at a value corresponding to maximum sustained peak or one-half peak, as outlined below.

If the total duration of the peak load exceeds 10 000 000 cycles during the expected life of the gear, use the value of this peak load for estimating gear size. If, however, the total duration of the peak load is less than 10 000 000 cycles, use one-half the peak load or the value of the highest sustained load, whichever is greater.

Given gear torque and the desired gear ratio, the charts give gear pitch diameter. The charts are based on case-hardened steel and should be used as follows:

1. For other materials, multiply the gear pitch diameter by the material factor from Table 34.1.
2. For general industrial gearing, the preliminary gear size is based on surface durability.
3. For straight-bevel gears, multiply the gear pitch diameter by 1.2; for Zerol bevel gears, multiply the gear pitch diameter by 1.3.
4. For high-capacity spiral-bevel and hypoid gears, the preliminary gear size is based on both surface capacity and bending strength. Choose the larger of the gear diameters, based on the durability chart and the strength chart.

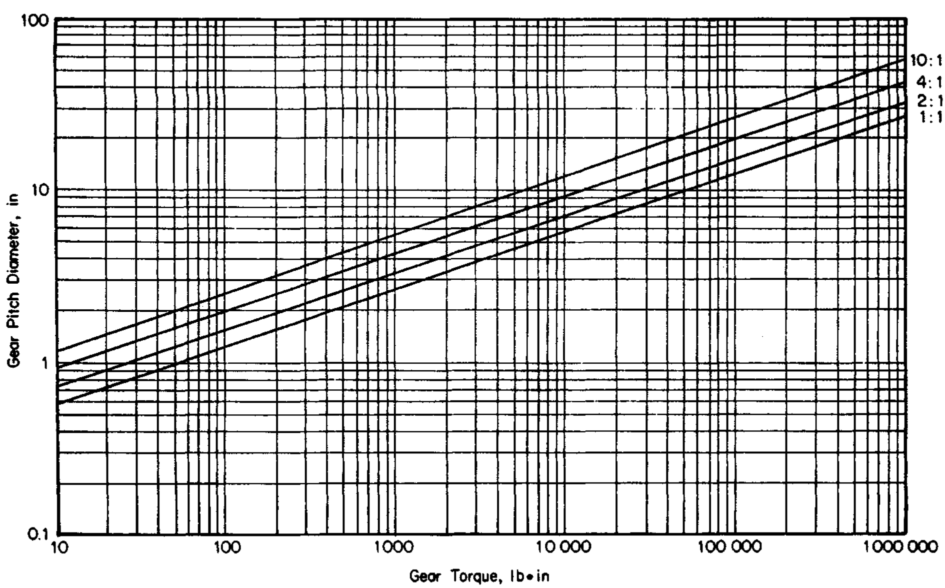

FIGURE 34.10 Gear pitch diameter based on surface durability.

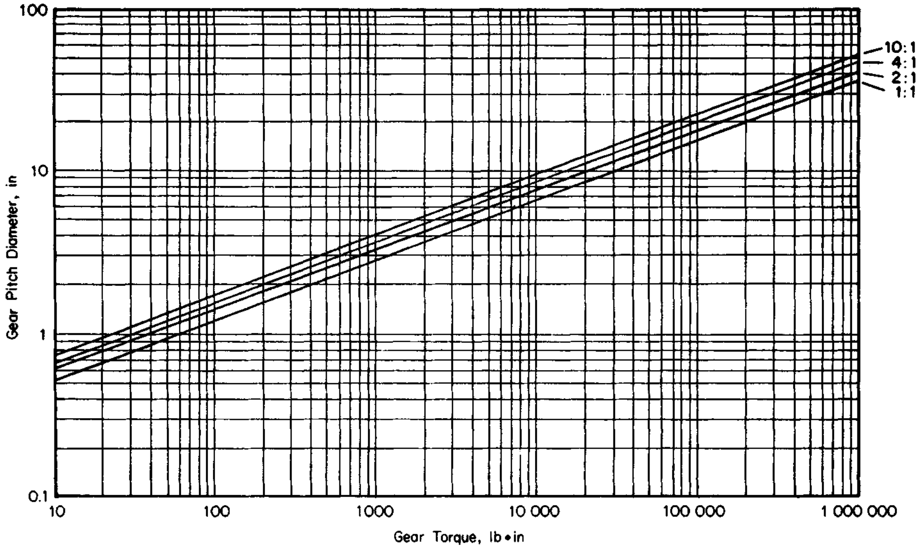

FIGURE 34.11 Gear pitch diameter based on bending strength.

5. For high-capacity ground spiral-bevel and hypoid gears, the gear diameter from the durability chart should be multiplied by 0.80.
6. For hypoid gears, multiply the gear pitch diameter by $D/(D + E)$.
7. Statically loaded gears should be designed for bending strength rather than surface durability. For statically loaded gears subject to vibration, multiply the gear diameter from the strength chart by 0.70. For statically loaded gears not subject to vibration, multiply the gear diameter from the strength chart by 0.60.
8. Estimated pinion diameter is $d = Dn/N$.

34.4.4 Number of Teeth

Figure 34.12 gives the recommended tooth numbers for spiral-bevel and hypoid gears. Figure 34.13 gives the recommended tooth numbers for straight-bevel and Zerol bevel gears. However, within limits, the selection of tooth numbers can be made in an arbitrary manner.

More uniform gears can be obtained in the lapping process if a common factor between gear and pinion teeth is avoided. Automotive gears are generally designed with fewer pinion teeth. Table 34.2 indicates recommended tooth numbers for automotive spiral-bevel and hypoid drives.

34.4.5 Face Width

The face width should not exceed 30 percent of the cone distance for straight-bevel, spiral-bevel, and hypoid gears and should not exceed 25 percent of the cone distance for Zerol bevel gears. In addition, it is recommended that the face width F be limited to

TABLE 34.1 Material Factors C_M

Gear		Pinion		Material factor C_M
Material	Hardness	Material	Hardness	
Case-hardened steel	58 R_C†	Case-hardened steel	60 R_C†	0.85‡
Case-hardened steel	55 R_C†	Case-hardened steel	55 R_C†	1.00
Flame-hardened steel	50 R_C†	Case-hardened steel	55 R_C†	1.05
Flame-hardened steel	50 R_C†	Flame-hardened steel	50 R_C†	1.05
Oil-hardened steel	375–425 H_B	Oil-hardened steel	375–425 H_B	1.20
Heat-treated steel	250–300 H_B	Case-hardened steel	55 R_C†	1.45
Heat-treated steel	210–245 H_B	Heat-treated steel	245–280 H_B	1.65
Cast iron		Case-hardened steel	55 R_C†	1.95
Cast iron		Flame-hardened steel	50 R_C†	2.00
Cast iron		Annealed steel	160–200 H_B	2.10
Cast iron		Cast iron		3.10

†Minimum values.
‡Gears must be file-hard.

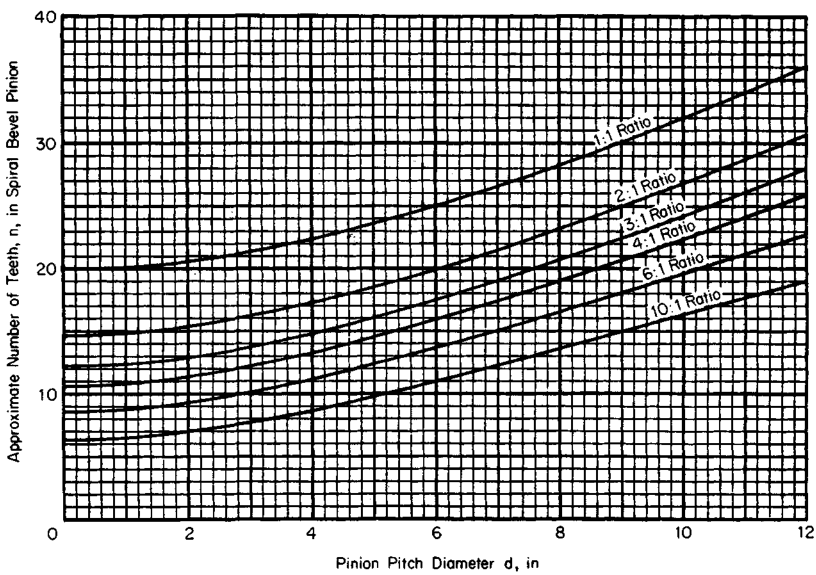

FIGURE 34.12 Recommended tooth numbers for spiral-bevel and hypoid gears.

$$F \leq \frac{10}{P_d}$$

The design chart in Fig. 34.14 gives the approximate face width for straight-bevel, spiral-bevel, and hypoid gears. For Zerol bevel gears, the face width given by this chart should be multiplied by 0.83.

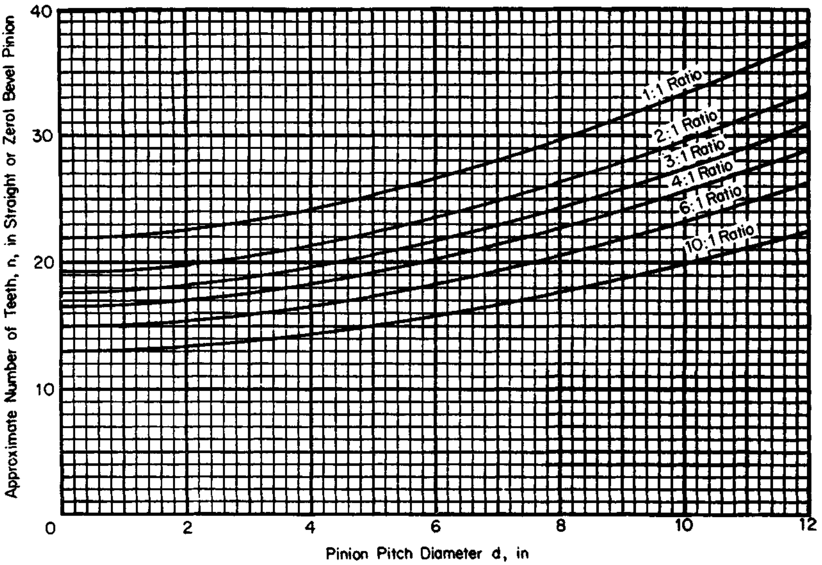

FIGURE 34.13 Recommended tooth numbers for straight- and Zerol-bevel gears.

34.4.6 Diametral Pitch

The diametral pitch is now calculated by dividing the number of teeth in the gear by the gear pitch diameter. Because tooling for bevel gears is not standardized according to pitch, it is not necessary that the diametral pitch be an integer.

TABLE 34.2 Recommended Tooth Numbers for Automotive Applications

Approximate ratio	Preferred no. pinion teeth	Allowable range
1.50/1.75	14	12 to 16
1.75/2.00	13	11 to 15
2.0/2.5	11	10 to 13
2.5/3.0	10	9 to 11
3.0/3.5	10	9 to 11
3.5/4.0	10	9 to 11
4.0/4.5	9	8 to 10
4.5/5.0	8	7 to 9
5.0/6.0	7	6 to 8
6.0/7.5	6	5 to 7
7.5/10.0	5	5 to 6

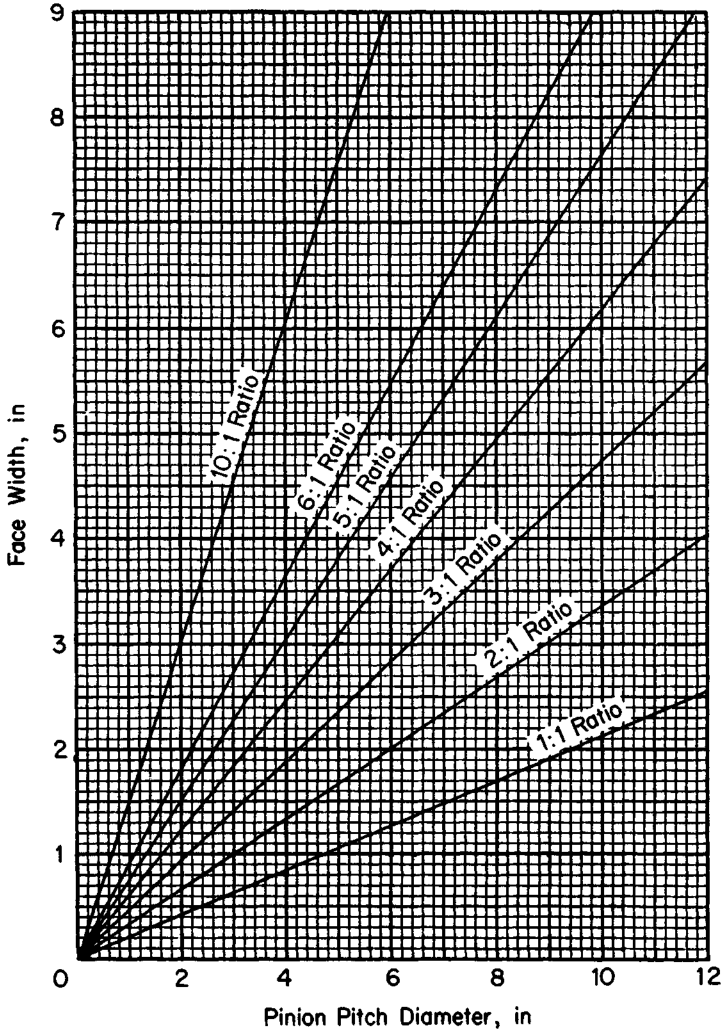

FIGURE 34.14 Face width of spiral-bevel and hypoid gears.

34.4.7 Hypoid Offset

In the design of hypoid gears, the offset is designated as being above or below center. Figure 34.15*a* and *b* illustrates the below-center position, and Fig. 34.15*c* and *d* illustrates the above-center position. In general, the shaft offset for power drives should not exceed 25 percent of the gear pitch diameter, and on very heavily loaded gears, the offset should be limited to 12.5 percent of the gear pitch diameter.

Hypoid pinions are larger in diameter than the corresponding spiral-bevel pinion. This increase in diameter may be as great as 30 percent, depending on the offset, spiral angle, and gear ratio.

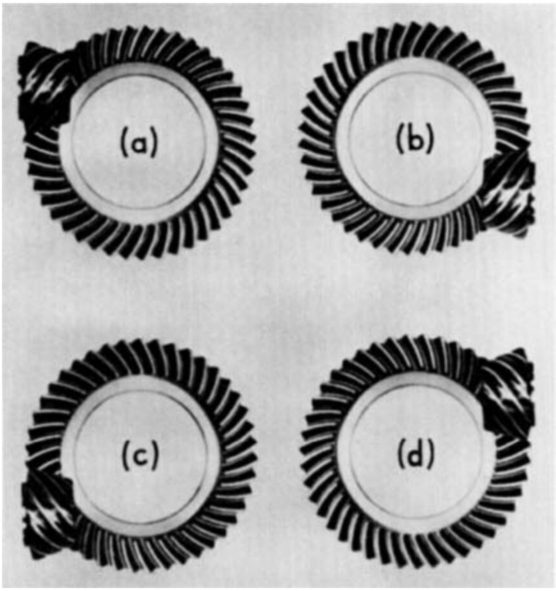

FIGURE 34.15 Hypoid offset. To determine the direction of offset, always look at the gear with the pinion at the right. Thus the gear sets of (*a*) and (*b*) are both offset *below* center; similar reasoning shows that (*c*) and (*d*) are offset *above* center. *(Gleason Machine Division.)*

34.4.8 Spiral Angle

In designing spiral-bevel gears, the spiral angle should be sufficient to give a face-contact ratio of at least 1.25. For maximum smoothness and quietness, the face-contact ratio should be between 1.50 and 2.00. High-speed applications should be designed with a face-contact ratio of 2.00 or higher for best results. Figure 34.16 may be used to assist in the selection of the spiral angle.

For hypoid gears, the desired pinion spiral angle can be calculated by

$$\psi_P = 25 + 5\sqrt{\frac{N}{n}} + 90\,\frac{E}{D}$$

where ψ_P is in degrees.

34.4.9 Pressure Angle

The commonly used pressure angle for bevel gears is 20°, although pressure angles of 22.5° and 25° are used for heavy-duty drives.

In the case of hypoids, the pressure angle is unbalanced on opposite sides of the gear teeth in order to produce equal contact ratios on the two sides. For this reason, the average pressure angle is specified for hypoids. For automotive drives, use 18° or 20°, and for heavy-duty drives, use 22.5° or 25°.

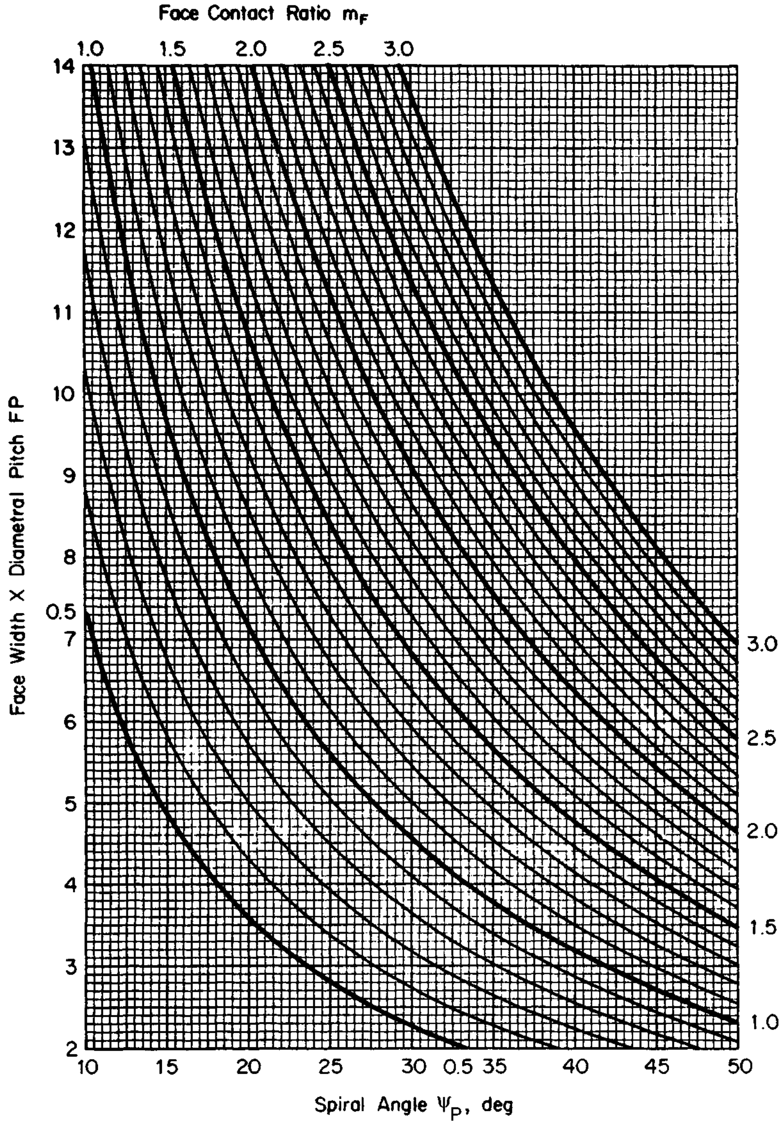

FIGURE 34.16 Selection of spiral angle.

34.4.10 Cutter Diameter

A cutter diameter must be selected for spiral-bevel, Zerol bevel, and hypoid gears. The usual practice is to use a cutter diameter approximately equal to the gear diameter. To increase adjustability of the gear set and obtain maximum strength, a smaller cutter should be used. Cutter diameters are standardized. Therefore, Table 34.3 is included to aid in cutter selection.

TABLE 34.3 Standard Cutter Radii Corresponding to Various Gear Pitch Diameters for Spiral-Bevel, Zerol Bevel, and Hypoid Gears

Pitch diameter D of gear, in	Cutter radius r_c, in (mm)
3.000–5.250	1.750
3.875–6.750	2.250
4.250–7.500	2.500
5.125–9.000	3.000
6.500–11.250	3.750
7.750–13.500	4.500
9.000–15.750	5.250
10.250–18.000	6.000
12.000–21.000	7.000
13.750–24.000	8.000
15.500–27.000	9.000
18.000–31.500	10.500
21.750–60.000	(320)
27.250–75.000	(400)
34.250–100.000	(500)

34.4.11 Materials and Heat Treatment

Through-hardening steels are used when medium wear resistance and medium load-carrying capacity are desired. The following steels are some of those used and listed, beginning with the steel of lowest hardenability: AISI 1045, 1144, 4640, 4150, and 4340. When greater hardenability is required for larger gears, it is sometimes necessary to increase the carbon content of these steels or to select a different steel.

Carburized gears are used when high wear resistance and high load-carrying capacity are required. Carburizing steels used in gears normally have a carbon content of 0.10 to 0.25 percent and should have sufficient alloy content to allow hardening in the section sizes in which they are used. For low-heat-treat distortion, 4620 or 8620 might be used; if high core hardness is desired, 9310 might be used. The following steels have been commonly used and are listed, beginning with the steel of lowest core hardenability: AISI 4620, 8620, 9310, and 4820.

Carburized gears should be specified as follows:

1. Total depth of carburized case after finishing operations
2. Surface hardness
3. Core hardness
4. Maximum case carbon content (optional)

Gears should be quenched from a temperature which will ensure a minimum amount of retained austenite.

Nitriding steels are used in applications which require high wear resistance with minimum distortion in heat treating. The commonly used steels are AISI 4140, 4150, and 4340. If extreme hardness and wear resistance are required, the nitralloy steels can be used. To achieve the desired results in the nitriding operation, all material should be hardened and tempered above the nitriding temperature prior to finish machining. Sharp corners should be avoided on external surfaces.

Nitrided gears should be specified as follows:

1. Total depth of nitrided case after finishing operations
2. Surface hardness
3. Core hardness

Cast iron is used in place of non-heat-treated steel where good wear resistance plus excellent machineability is required. Complicated shapes can be cast more easily from iron than they can be produced by machining from bars or forgings.

34.5 GEAR-TOOTH DIMENSIONS

34.5.1 Calculation of Basic Bevel-Gear-Tooth Dimensions

All bevel-gear-tooth dimensions are calculated in a similar manner. Therefore, straight-bevel, spiral-bevel, and Zerol bevel gears are considered as a group. In Sec. 34.4 we selected

1. Number of pinion teeth n
2. Number of gear teeth N
3. Diametral pitch P_d
4. Shaft angle Σ
5. Face width F
6. Pressure angle ϕ
7. Spiral angle ψ
8. Hand of spiral (pinion), left-hand/right-hand (LH/RH)
9. Cutter radius r_c

The formulas in Table 34.4 are now used to calculate the blank and tooth dimensions.

34.5.2 Tooth Taper

Spiral-bevel- and hypoid-gear blanks are designed by one of four methods—standard taper, duplex taper, tilted root line, or uniform depth.

Standard taper is the case where the root lines of mating members, if extended, would intersect the pitch-cone apex. The tooth depth changes in proportion to the cone distance.

Duplex taper is the case where the root lines are tilted so that the slot width is constant. This condition permits each member of a pair to be finished in one operation, using circular cutters which cut in one slot.

Tilted root-line taper is a compromise between duplex and standard taper. The blanks are designed with duplex taper except when the taper becomes excessive. When the taper is 1.3 times standard taper or greater, 1.3 times standard taper is used.

Uniform-depth taper is the case where the root lines are not tilted. The tooth depth is uniform from the inside to the outside of the blank.

TABLE 34.4 Formulas for Computing Blank and Tooth Dimensions

Item	Item no.	Member	Formula
Pitch diameter	1	Pinion Gear	$d = \dfrac{n}{P_d}$ $D = \dfrac{N}{P_d}$
Pitch angle	2	Pinion Gear	$\gamma = \tan^{-1} \dfrac{\sin \Sigma}{N/n + \cos \Sigma}$ $\Gamma = \Sigma - \gamma$
Outer cone distance	3	Both	$A_o = \dfrac{0.50D}{\sin \Gamma}$
Mean cone distance	4	Both	$A_m = A_o - 0.5F$
Depth factor k_1	5	Both	Table 34.5
Mean working depth	6	Both	$h = \dfrac{k_1 A_m}{P_d A_o} \cos \psi$
Clearance factor k_2	7	Both	Table 34.6
Clearance	8	Both	$c = k_2 h$
Mean whole depth	9	Both	$h_m = h + c$
Equivalent 90° ratio	10	Both	$m_{90} = \sqrt{\dfrac{N \cos \gamma}{n \cos \Gamma}}$
Mean addendum factor C_1	11	Both	Table 34.7
Mean circular pitch	12	Both	$P_m = \dfrac{\pi A_m}{P_d A_o}$
Mean addendum	13	Pinion Gear	$a_P = h - a_G$ $a_G = C_1 h$
Mean dedendum	14	Pinion Gear	$b_P = h_m - a_P$ $b_G = h_m - a_G$
Sum of dedendum angles	15	Both	$\Sigma\delta$ (see Sec. 34.5.2)

TABLE 34.4 Formulas for Computing Blank and Tooth Dimensions (*Continued*)

Item	Item no.	Member	Formula
Dedendum angle	16	Pinion Gear	δ_P (see Sec. 34.5.2) δ_G (see Sec. 34.5.2)
Face angle of blank	17	Pinion Gear	$\gamma_o = \gamma + \delta_G$ $\Gamma_o = \Gamma + \delta_P$
Root angle of blank	18	Pinion Gear	$\gamma_R = \gamma - \delta_P$ $\Gamma_R = \Gamma - \delta_G$
Outer addendum	19	Pinion Gear	$a_{oP} = a_P + 0.5F \tan \delta_G$ $a_{oG} = a_G + 0.5F \tan \delta_P$
Outer dedendum	20	Pinion Gear	$b_{oP} = b_P + 0.5F \tan \delta_P$ $b_{oG} = b_G + 0.5F \tan \delta_G$
Outer working depth	21	Both	$h_k = a_{oP} + a_{oG}$
Outer whole depth	22	Both	$h_t = a_{oP} + b_{oP}$
Outside diameter	23	Pinion Gear	$d_o = d + 2a_{oP} \cos \gamma$ $D_o = D + 2a_{oG} \cos \Gamma$
Pitch apex to crown	24	Pinion Gear	$x_o = A_o \cos \gamma - a_{oP} \sin \gamma$ $X_o = A_o \cos \Gamma - a_{oG} \sin \Gamma$
Mean diametral pitch	25	Both	$P_{dm} = P_d \dfrac{A_o}{A_m}$
Mean pitch diameter	26	Pinion Gear	$d_m = \dfrac{n}{P_{dm}}$ $D_m = \dfrac{N}{P_{dm}}$
Thickness factor K	27	Both	Fig. 34.17
Mean normal circular thickness	28	Pinion Gear	$t_n = P_m \cos \psi - T_n$ $T_n = \dfrac{P_m}{2 \cos \psi} - (a_P - a_G) \tan \phi + \dfrac{K \cos \psi}{P_{dm} \tan \phi}$

TABLE 34.4 Formulas for Computing Blank and Tooth Dimensions (*Concluded*)

Item	Item no.	Member	Formula
Outer normal backlash allowance	29	Both	B (Table 34.8)
Mean normal chordal thickness	30	Pinion	$t_{nc} = t_n - \dfrac{t_n^3}{6d_m^2} - 0.5B\dfrac{A_m}{A_o}\sec\phi$
		Gear	$T_{nc} = T_n - \dfrac{T_n}{6D_m^2} - 0.5B\left(\dfrac{A_m}{A_o}\right)\sec\phi$
Mean chordal addendum	31	Pinion	$a_{cP} = a_P + \dfrac{t_n^2\cos\gamma}{4d_m}$
		Gear	$a_{cG} = a_G + \dfrac{T_n^2\cos\Gamma}{4D_m}$

TABLE 34.5 Depth Factor

Type of gear	No. pinion teeth	Depth factor k_1
Straight bevel	12 and higher	2.000
Spiral bevel	12 and higher	2.000
	11	1.995
	10	1.975
	9	1.940
	8	1.895
	7	1.835
	6	1.765
Zerol bevel	13 and higher	2.000
Hypoid	11 and higher	4.000
	10	3.900
	9	3.8
	8	3.7
	7	3.6
	6	3.5

TABLE 34.6 Clearance Factors

Type of gear	Clearance factor k_2
Straight bevel	0.140
Spiral bevel	0.125
Zerol bevel	0.110
Hypoid	0.150

TABLE 34.7 Mean Addendum Factor

Type of gear	No. pinion teeth	Mean addendum factor C_1
Straight bevel	12 and higher	C_1†
Spiral bevel	12 and higher	C_1†
	11	0.490
	10	0.435
	9	0.380
	8	0.325
	7	0.270
	6	0.215
Zerol bevel	13 and higher	C_1†
Hypoid	21 and higher	C_1†
	9 to 20	0.170
	8	0.150
	7	0.130
	6	0.110

†Use $C_1 = 0.270 + 0.230/(m_{90})^2$.

TABLE 34.8 Minimum Normal Backlash Allowance†

Range of diametral pitch, teeth/in	Allowance, in (for AGMA quality number range) 4 to 9	10 to 13
1.00–1.25	0.032	0.024
1.25–1.50	0.027	0.020
1.50–2.00	0.020	0.015
2.00–2.50	0.016	0.012
2.50–3.00	0.013	0.010
3.00–4.00	0.010	0.008
4.00–5.00	0.008	0.006
5.00–6.00	0.006	0.005
6.00–8.00	0.005	0.004
8.00–10.00	0.004	0.003
10.00–12.00	0.003	0.002
12.00–16.00	0.003	0.002
16.00–20.00	0.002	0.001
20.00–25.00	0.002	0.001

†Measured at outer cone in inches.

In many cases, the type of taper depends on the manufacturing method. Before selecting a tooth taper, you should consult with the manufacturer to ensure compatibility between the design and the cutting method.

Straight-bevel gears are usually designed with standard taper. Zerol bevel gears are usually designed with duplex taper.

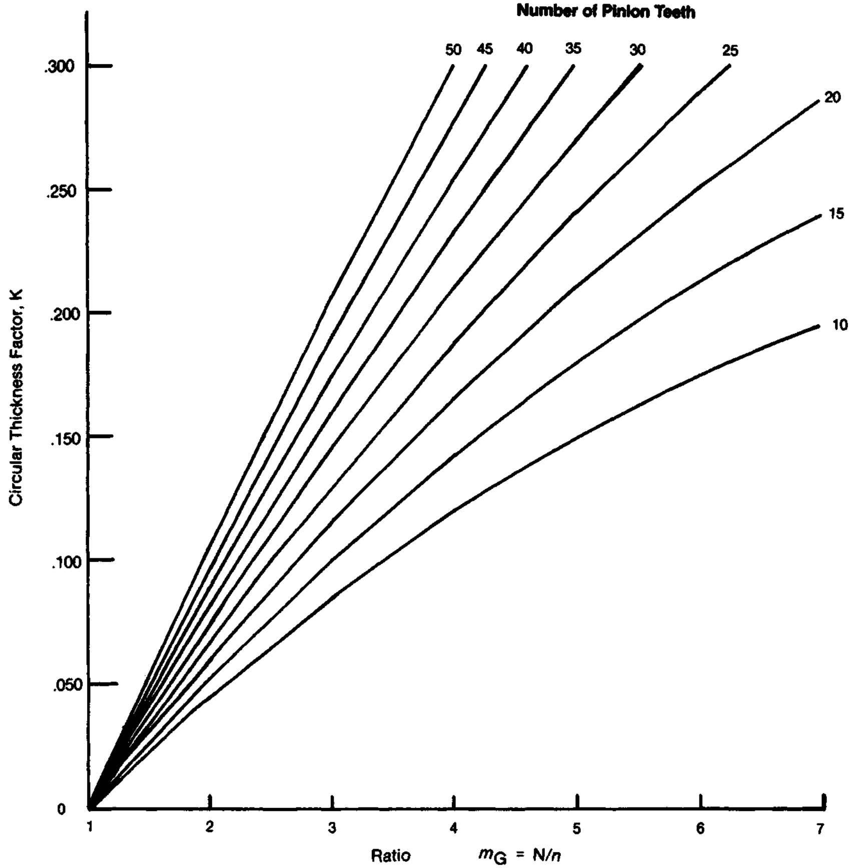

FIGURE 34.17 Circular thickness factor. These curves are plotted from the equation $K = -0.088 + 0.092m_G - 0.004m_G^2 + 0.0016\,(n - 30)\,(m_G - 1)$.

The formulas used to calculate the sum of dedendum angles and the dedendum angles are shown in Table 34.9.

34.5.3 Hypoid Dimensions

The geometry of hypoid gears is complicated by the offset between the axes of the mating members. Therefore a separate set of calculation formulas is needed.

The starting data are the same as for bevel gears with the following exceptions:

1. Hypoid offset E is required.
2. Pinion spiral angle ψ_P is specified.

The formulas in Table 34.10 are now used to calculate the blank and tooth dimensions.

TABLE 34.9 Formulas for Computing Dedendum Angles and Their Sum

Type of taper	Formula
Standard	$\Sigma\delta = \tan^{-1}\dfrac{b_P}{A_m} + \tan^{-1}\dfrac{b_G}{A_m}$ $\delta_P = \tan^{-1}\dfrac{b_P}{A_m} \qquad \delta_G = \Sigma\delta - \delta_P$
Duplex	$\Sigma\delta = \dfrac{90[1 - (A_m/r_c)\sin\psi]}{(P_d A_o \tan\phi\cos\psi)}$ $\delta_P = \dfrac{a_G}{h}\Sigma\delta \qquad \delta_G = \Sigma\delta - \delta_P$
Tilted root line	Use $\Sigma\delta = \dfrac{90[1 - (A_m/r_c)\sin\psi]}{(P_d A_o \tan\phi\cos\psi)}$ or $= 1.3\tan^{-1}\dfrac{b_P}{A_m} + 1.3\tan^{-1}\dfrac{b_G}{A_m}$ whichever is smaller. $\delta_P = \dfrac{a_G}{h} \qquad \delta_G = \Sigma\delta - \delta_P$
Uniform depth	$\Sigma\delta = 0$ $\delta_P = \delta_G = 0$

34.5.4 AGMA References†

The following AGMA standards are helpful in designing bevel and hypoid gears:

AGMA Design Manual for Bevel Gears, 2005
AGMA Rating Standard for Bevel Gears, 2003

These are available through American Gear Manufacturer's Association, 1500 King Street, Suite 201, Alexandria, VA 22314-2730.

34.6 GEAR STRENGTH

Under ideal conditions of operation, bevel and hypoid gears have a tooth contact which utilizes the full working profile of the tooth without load concentration in any

† The notation and units used in this chapter are the same as those used in the AGMA standards. These may differ in some respects from those used in other chapters of this Handbook.